THE IGNITION OF A CATALYTIC SURFACE

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The Shvets method [1] is used to study the ignition of a reacting gas from a heated catalytic surface in the presence of conductive and convective heat transfer from that heated surface.

Ribo and Valentin [2] observed experimentally that there exist two ignition regimes for a catalytic surface. At a comparatively low temperature for the catalytic surface (for a reacting system hydrogen-air-platinum $T_C \le 480°$ K, we have heterogeneous combustion in the molecules of the reacting substance. At a high temperature for the heated surface ($T_C \ge$ $\ge 900°$ for the same system [2]), a homogeneous reaction is essential. Spalding [3] provided a detailed review of the literature on the transfer of heat in chemically reacting gases and indicated the possible practical applications.

We will study theoretically the ignition of a catalytic surface for the case of a homogeneous reaction of the first order.

For conductive heat transfer from a heated surface the problem reduces mathematically to the solution of the system of equations

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{\partial \theta}{\partial \tau} - c \exp \theta, \tag{1}$$

$$\frac{\partial^2 c}{\partial z^2} = L \left(\frac{\partial c}{\partial \tau} + \gamma c \exp \theta \right) \tag{2}$$

having the boundary and initial conditions

$$\theta(0, \tau) = 0, \ \theta(z, 0) = \theta(\infty, \tau) = -\theta_0, \quad c(z, 0) = c(\infty, \tau) = 1. \tag{3}$$

In deriving Eqs. (1) and (2) we used the Frank-Kamenetskii [4] approximation for $\exp(-E/RT)$, while in the derivation of the boundary and initial conditions (3) we assumed that the concentration of the reagent at the heated surface instantaneously drops to zero. This is explained by the fact that the rate of the chemical reaction at the catalytic surface is very great [5] and is in agreement with the experimental data of [2]. It was also assumed that the thermophysical coefficients $\lambda = \lambda_0 T/T_0$, and $\rho = \rho_0 T_0/T$ and the Lewis-Semenov number does not vary with a change in temperature, and it was assumed that the transformation of the independent variable $\bar{z} = \int_0^{z_1} \frac{\rho}{\rho_0} dz_1$, which is analogous to the Dorodnitsyn transformation [6].

Since the rate of the chemical reaction is an exponential function of temperature, the reaction takes place within narrow temperature and diffusion boundary layers. Having introduced the thicknesses of the temperature and diffusion boundary layers, we bring the boundary and initial conditions (3) to the form

$$\theta(0, \tau) = 0, \ \theta(\Delta_1, \tau) = -\theta_0, \ c(0, \tau) = 0, \quad c(\Delta_2, \tau) = 1, \ \Delta_1(0) = \Delta_2(0) = 0. \tag{4}$$

We use the Shvets method [1] to solve the boundary problem (1), (2), and (4). As first approximations for the temperature and concentration we obtain

$$\theta_1 = -\theta_0 z/\Delta_1, \quad c_1 = z/\Delta_2. \tag{5}$$

Substituting (5) into the right-hand parts of (1) and (2) and integrating the result of the substitution twice with respect to z and with consideration of (4), we obtain the second approximations

$$\theta_2 = \frac{\theta_0 z^3}{6\Delta_1^2} \frac{d\Delta_1}{d\tau} + \frac{\Delta_1^3}{\theta_0^3 \Delta_2} \left[2 - \left(2 + \frac{\theta_0 z}{\Delta_1} \right) \exp - \frac{\theta_0 z}{\Delta_1} \right] + Az, \tag{6}$$

$$c_2 = -\frac{L z^3}{6\Delta_2^2} \frac{d\Delta_2}{d\tau} - \frac{\gamma L \Delta_1^3}{\theta_0^3 \Delta_2} \times \left[2 - \left(2 + \frac{\theta_0 z}{\Delta_1} \right) \exp - \frac{\theta_0 z}{\Delta_1} \right] + Bz. \tag{7}$$

For simplicity, subsequently we will drop the terms containing the factor $\varepsilon = \exp -\theta_0$, since when $\theta_0 \gg 1$ the quantity $\varepsilon \ll 1$.

Having satisfied (6) and (7) in terms of the Shvets conditions [1], we derive the differential equations for the determination of the quantities Δ_1 and Δ_2:

$$\Delta_1 \frac{d\Delta_1}{d\tau} = 3 + \frac{6\Delta_1^3}{\theta_0^4 \Delta_2}, \quad L \Delta_2 \frac{d\Delta_2}{d\tau} = 3 + \frac{6\gamma L \Delta_1^3}{\theta_0^3 \Delta_2}. \tag{8}$$

Then, by means of the method of perturbations [7] for moderate values of τ we have

$$\Delta_1^2 = 6\tau \left(1 + \frac{6\tau L^{1/2}}{\theta_0^4} \right), \quad \Delta_2^2 = \frac{6\tau}{L} \left(1 + \frac{6\gamma L^{3/2} \tau}{\theta_0^3} \right). \tag{9}$$

In deriving (9) we took into consideration the initial conditions (4) and dropped all powers of the perturbations above the first. We note that the first terms in expressions (9) yield values for the thicknesses of the temperature and diffusion boundary layers of a nonreacting gas that are exact within the scope of the second approximation according to the Shvets method.

From the Zel'dovich condition [8] $d\theta/dz|_{z=0} = 0$, which is an approximate condition of ignition, we find the heating time

$$\tau_* = \frac{\theta_0^3}{4L^{1/2}}\left[1 + \frac{3}{4\theta_0}(1 + \gamma\theta_0 L)\right]. \qquad (10)$$

We see from expressions (6), (7), and (9) that when $0 < \tau \leq \tau_*$, the thicknesses of the boundary layers and the profiles of the temperature and concentration differ little from the corresponding quantities found without consideration of the liberation of heat in the homogeneous reaction. Since the pronounced convergence of the successive approximations of the Shvets method for an extensive class of linear and self-similar problems is shown in [1] and demonstrated theoretically by Gandin [9], in view of the smallness of the perturbations due to the chemical reaction it is possible to hold that the convergence of the Shvets method applies also to our case, at least when $0 < \tau \leq \tau_*$. This is also valid for two other problems, considered below.

By means of (9) and (10) we find the thickness of the heated layer

$$\Delta_{1*} = \Delta_1(\tau_*) = \frac{\sqrt{6}\,\theta_0^{3/2}}{2L^{1/4}}\left[1 + \frac{3}{8\theta_0}(3 + \gamma\theta_0 L)\right]. \qquad (11)$$

The quantity of heat transferred by the heated plate is equal to

$$Q_* = -\int_0^{\tau_*} \frac{\partial\theta_2}{\partial z}\bigg|_{z=0} d\tau =$$

$$= \frac{\theta_0^{5/2}}{\sqrt{6}\,L^{1/4}}\left[1 + \frac{9}{40\theta_0}(3 + \gamma\theta_0 L)\right]. \qquad (12)$$

We see from formulas (10), (11), and (12) that with an increasing Lewis-Semenov number the heating time diminishes as $L^{-1/2}$, while Δ_{1*} and Q_* diminish as $L^{-1/4}$.

Thus, with all other conditions equal, the ignition of the reacting gas is all the easier, the larger the coefficient of heat diffusion in comparison with the coefficient D of concentrated diffusion.

On ignition of a noncatalytic surface in the absence of complete reagent burning, with the Shvets method [1] we easily find

$$\tau_* = \frac{\theta_0^2}{4}\left(1 + \frac{8}{4\theta_0} + \frac{3}{8\theta_0^2}\right), \qquad (13)$$

$$\Delta_{1*} = \frac{\sqrt{6}\,\theta_0}{2}\left(1 + \frac{3}{4\theta_0} + \frac{9}{32\theta_0^2}\right), \qquad (14)$$

$$Q_* = \frac{\theta_0^2}{\sqrt{6}}\left(1 + \frac{6}{5\theta_0}\right). \qquad (15)$$

Formulas (13) and (14) follow from (7) and (8) of [10] when e = 1. Comparison of formulas (13)–(15) with the corresponding formulas derived with a computer [11, 12], has demonstrated that they differ little from the exact. In particular, expression (13) for $\theta_0 \geq 10$ yields numerical data virtually coincident when $\beta = 0$ with the numerical data of reference [12], whose error does not exceed 8%. Consequently, the error in (13) is approximately equal to 8%, which coincides, as was to be expected from the above-cited considerations regarding convergence of the Shvets method, with the accuracy of the second approximation for the temperature of the corresponding linear boundary problem [1]. Formulas (13)–(15) retain their accuracy even in the presence of complete combustion, if $\gamma\theta_0 \ll 1$, since in this case, according to the results of [12], the complete combustion has little effect on the characteristics of ignition.

Having compared (10)–(12) with (13)–(15) when L = = 1 and $\gamma\theta_0 \ll 1$, for the ignition of the catalytic surface, we find that τ_* exceeds the heating time (13) by a factor of θ_0, while Δ_{1*} and Q_* are greater than the thickness of the heated layer (14) and the quantity of transferred heat (15) by a factor of $(\theta_0)^{1/2}$.

Thus, to derive an identical heating time, all other conditions being equal, we must heat the catalytic surface to a higher temperature and transfer a greater quantity of heat from the heated surface; i. e., the catalytic surface makes ignition of the reacting gas more difficult. This fact is in agreement with experimental data [13] and the qualitative considerations of [5].

In convective heat transfer from the heated catalytic surface we find that the problem of the ignition of the reacting gas reduces to the solution of a system of equations from hydromechanics, heat conduction and diffusion:

$$\frac{\partial^2 u}{\partial y^2} = u\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy, \qquad (16)$$

$$\frac{\partial^2 \theta}{\partial y^2} = \Pr\left(u\frac{\partial\theta}{\partial x} - \frac{\partial\theta}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy\right) - \alpha c\exp\theta, \qquad (17)$$

$$\frac{\partial^2 c}{\partial y^2} = \mathrm{Sc}\left(u\frac{\partial c}{\partial x} - \frac{\partial c}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy + \frac{\alpha\gamma c}{\Pr}\exp\theta\right). \qquad (18)$$

In deriving system (16)–(18) we assumed that $\mu = = \mu_0 T/T_0$, $\rho = \rho_0 T_0/T$ and $\lambda = \lambda_0 T/T_0$, and as in [10] we used the Dorodnitsyn transformation [6]. As in [10], having introduced the thickness of the hydrodynamic, temperature, and diffusion boundary layers, we write the boundary and initial conditions for system (16)–(18) in the form:

$$\begin{gathered} u(x,\ 0) = 0, \quad u(x,\ \Delta) = 1, \\ \theta(x,\ 0) = 0, \quad \theta(x,\ \Delta_1) = -\theta_0, \\ c(x,\ 0) = 0, \quad c(x,\ \Delta_2) = 1, \\ \Delta(0) = \Delta_1(0) = \Delta_2(0) = 0. \end{gathered} \qquad (19)$$

To solve the boundary problem (16)–(19) we use the modified Shvets method [14]. The hydrodynamic part o of the problem has been solved in [14], where $\Delta = = 4(x)^{1/2}$ was found, as was the velocity profile u(x, y). Let us solve the equation of heat conduction and diffusion in a manner analogous to the way this was done in [10] and as done above for the case of conductive heat transfer. Dropping the intermediate calculations, we write the final results:

$$\theta_2 = \frac{\Pr \theta_0 y^4}{48\sqrt{x}\Delta_1}\left(\Delta_1^{-1}\frac{d\Delta_1}{dx} - \frac{1}{4x}\right) + \\ + \frac{\alpha\Delta_1^3}{\theta_0^3\Delta_2}\left[2 - \left(2 + \frac{\theta_0 y}{\Delta_1}\right)\exp\left(-\frac{\theta_0 y}{\Delta_1}\right)\right] + Cy, \quad (20)$$

$$c_2 = -\frac{\mathrm{Sc}\, y^4}{48\sqrt{x}\Delta_2}\left(\Delta_2^{-1}\frac{d\Delta_2}{dx} - \frac{1}{4x}\right) - \\ - \frac{\alpha\gamma\,\mathrm{Sc}\,\Delta_1^3}{\Pr\Delta_2\theta_0^3}\left[2 - \left(2 + \frac{\theta_0 y}{\Delta_1}\right)\exp\left(-\frac{\theta_0 y}{\Delta_1}\right)\right] + Fy, \quad (21)$$

$$\Delta_1^3 = \frac{64x^{3/2}}{\Pr}\left(1 + \frac{96\alpha\,\mathrm{Sc}^{1/3}x}{7\Pr\theta_0^4}\right),$$

$$\Delta_2^3 = \frac{64x^{3/2}}{\mathrm{Sc}}\left(1 + \frac{96\alpha\gamma\,\mathrm{L}\mathrm{Sc}^{1/3}x}{7\Pr\theta_0^3}\right). \quad (22)$$

The distance $x = x_*$, at which the approximate Zel'dovich [8] ignition condition is satisfied, is equal to

$$x_* = \frac{\Pr\mathrm{Sc}^{-1/3}\theta_0^4}{4\alpha(3\theta_0 - 8)}\left[1 - \frac{8}{7(3\theta_0 - 8)}(3 - \gamma\theta_0\mathrm{L})\right], \quad (23)$$

while the formula for the thickness of the heated layer has the form

$$\Delta_{1*} = \Delta_1(x_*) = \frac{4\sqrt{x_*}}{\Pr^{1/3}}\left[1 + \frac{8}{7(3\theta_0 - 8)}\right]. \quad (24)$$

We see from formula (24) that, for the nonreacting gas, when $0 < x \le x_*$ the thickness of the temperature boundary layer differs little, for $\theta_0 \gg 1$, from the thickness of the temperature boundary layer [14]. Comparing x_* for $\Pr \approx \mathrm{Sc} \approx 1$ and $\gamma\theta_0 \ll 1$, which is the case for many reacting gases, with formula (20) of [10], we see that, on ignition of the catalytic surface, x_* is approximately greater by a factor of θ_0 than the quantity x_* determined there. So great a quantitative effect cannot be explained by the complete combustion of the reagent. Comparison of (22) of [10] with the numerical data derived by means of the computer [15] demonstrated that even in a reaction of the second order the complete combustion of the reagent increases the magnitude of x_* only by 30%. The factor responsible for the considerable exaggeration of x_* is the fact that the catalytic surface serves as a sink for the reacting substance.

In conclusion we note that there are no fundamental difficulties in determining x_* or the other ignition characteristics by the modified Shvets method [14] for any surfaces of rotation; however, the formulas and calculations are more cumbersome.

Let us examine the ignition of a reacting gas on a heated catalytic surface in the case of free convection. Physically this problem is formulated as in [10] for the ignition of a reacting gas on a noncatalytic surface. As in [10], we also assume that the thermophysical coefficients are constant, and that in the hydrodynamic sense the reacting gas is incompressible. Mathematically, the problem reduces to the solution of the system of equations:

$$\frac{\partial^2 U}{\partial\eta^2} = \left(U\frac{\partial U}{\partial\xi} - \frac{\partial U}{\partial\eta}\int_0^\eta \frac{\partial U}{\partial\xi}d\eta\right) - \theta_0 - \theta, \quad (25)$$

$$\frac{\partial^2\theta}{\partial\eta^2} = \Pr\left(U\frac{\partial\theta}{\partial\xi} - \frac{\partial\theta}{\partial\eta}\int_0^\eta \frac{\partial U}{\partial\xi}d\eta\right) - ac\exp\theta, \quad (26)$$

$$\frac{\partial^2 c}{\partial\eta^2} = \mathrm{Sc}\left(U\frac{\partial c}{\partial\xi} - \frac{\partial c}{\partial\eta}\int_0^\eta \frac{\partial U}{\partial\xi}d\eta + \frac{a\gamma c}{\Pr}\exp\theta\right). \quad (27)$$

Having introduced the thicknesses of the thermal and diffusion boundary layers, we write the boundary and initial conditions in the form:

$$U(\xi, 0) = U(\xi, \Delta_1) = 0, \quad \theta(\xi, 0) = 0, \\ \theta(\xi, \Delta_1) = -\theta_0, \quad c(\xi, 0) = 0, \quad c(\xi, \Delta_2) = 1, \\ \Delta_1(0) = \Delta_2(0) = 0. \quad (28)$$

We solve the boundary problem (25)–(28) by the Shvets method [1]. Omitting the intermediate calculations, we proceed at once to write the final results:

$$\theta_2 = \frac{\Pr\theta_0^2\eta^4}{8\Delta_1}\left(\frac{\eta^2}{30\Delta_1^2} - \frac{\eta}{5\Delta_1} + \frac{1}{3}\right)\frac{d\Delta_1}{d\xi} + \\ + \frac{a\Delta_1^3}{\Delta_2\theta_0^3}\left[2 - \left(2 + \frac{\theta_0\eta}{\Delta_1}\right)\exp\left(-\frac{\theta_0\eta}{\Delta_1}\right)\right] + G\eta, \quad (29)$$

$$c_2 = -\frac{\mathrm{Sc}\,\theta_0\eta^4}{4\Delta_2}\times \\ \times\left[\frac{\eta^2}{45\Delta_1}\left(\Delta_2^{-1}\frac{d\Delta_2}{d\xi} - \frac{\Delta_1^{-1}}{4}\frac{d\Delta_1}{d\xi}\right) - \right. \\ - \frac{\eta}{10\Delta_2}\frac{d\Delta_2}{d\xi} + \frac{1}{9}\times \\ \left.\times\left(\frac{\Delta_1}{\Delta_2}\frac{d\Delta_2}{d\xi} + \frac{1}{2}\frac{d\Delta_1}{d\xi}\right)\right] - \frac{a\gamma\,\mathrm{Sc}\,\Delta_1^3}{\Pr\Delta_2\theta_0^3}\times \\ \times\left[2 - \left(2 + \frac{\theta_0\eta}{\Delta_1}\right)\exp\left(-\frac{\theta_0\eta}{\Delta_1}\right)\right] + H\eta, \quad (30)$$

$$\Delta_1 = \alpha_1\xi^{1/4} + \alpha_2\xi^{1/2} + \alpha_3\xi^{3/4} + \ldots, \\ \Delta_2 = \beta_1\xi^{1/4} + \beta_2\xi^{1/2} + \beta_3\xi^{3/4} + \ldots, \quad (31)$$

$$\xi_* = \frac{256\,\theta_0^8\beta_1^2}{a^2\alpha_1^6(11\theta_0 - 32)^2}. \quad (32)$$

Unlike the previous cases, here the differential equations for the determination of Δ_1 and Δ_2 are solved by the method of expansion in series, as a result of which we found $\alpha_1 = \left(\frac{960}{11\Pr\theta_0}\right)^{1/4}$, $\alpha_2 = \beta_2 = 0$, and $\alpha_3 = \frac{320a}{11P\beta_1\theta_0^5}$, where β_1 is defined by the equation

$$\beta_1^4 = \left[\frac{\mathrm{Sc}\,\theta_0}{8}\left(\frac{\beta_1}{24\alpha_1} - \frac{1}{5} + \frac{\alpha_1}{8\beta_1}\right)\right]^{-1} \quad (33)$$

This equation can be solved by the method of iterations [16]. When $\Pr = \mathrm{Sc}$ from (33) we obtain $\beta_1 = \alpha_1$. The first terms in (31) yield values of Δ_1 and Δ_2 for nonreacting gases, within the framework of the second approximation of Shvets.

In deriving the formulas for ξ_*, where ξ_* is the distance from the edge of the plate at which the Zel'dovich [8] ignition condition is initially satisfied, we left only the principal term of the expansion of ξ_* in powers of θ_0^{-1}. Knowing ξ_*, we easily find that when $0 < \xi \leq \xi_*$ the thicknesses Δ_1 and Δ_2 differ little from the corresponding quantities for nonreacting gases.

If we select the maximum velocity $U = U_{1m}$ of the liquid particle as the characteristic velocity, then, knowing ξ_*, we can easily find the minimum time of thermal contact between the reacting gas and the heated surface—the time required for the ignition:

$$\tau_* = \frac{a}{\mathrm{Pr}} \int\limits_0^{\xi_*} \frac{d\xi}{U_{1m}} = \frac{288\sqrt{3}\,\theta_0^2\beta_1}{11\,\mathrm{Pr}\,\alpha_1^5}\left(1 + \frac{80}{33\,\theta_0}\right). \quad (34)$$

The quantity $U_{1m} = \theta_0\Delta_1^2/9(3)^{1/2}$ was found as a maximum of the function $U_1(\xi, \eta)$ determined from the second of the formulas of (30) in [10], while the thickness of the thermal boundary layer was determined from the first of the formulas of (31). For the ignition of a noncatalytic surface, by means of formulas (30) and (36) of [10] we obtain

$$\tau_* = \frac{33\sqrt{3}\,\theta_0^3}{160} \ln\left(1 + \frac{16}{11\theta_0 - 16}\right) \approx$$

$$\approx 0.3\sqrt{3}\,\theta_0^2\left(1 + \frac{8}{11\theta_0}\right). \quad (35)$$

Comparison of (10) and (35) demonstrates that τ_* for free convection is greater than τ_* for conductive heat transfer by a factor of approximately two, which is apparently explained by the lower rate of heat transfer from the heated surface to the reacting gas in the case of free convection.

Comparing (34) with formula (35) for $\mathrm{Pr} = \mathrm{Sc} = 1$ and $\theta_0 \gg 1$, we see that τ_* on ignition of the catalytic surface, all other conditions being equal, is greater than the magnitude of (35) by a factor of θ_0, since from formula (34) when $\mathrm{Pr} = \mathrm{Sc} = 1$ we have $\tau_* \approx 0.3(3)^{1/2}\theta_0^3$.

If instead of the Frank-Kamenetskii approximation [4] for $\exp(-E/RT)$ we use the more exact approximation

$$\exp\frac{\theta}{1+\beta\theta} \approx \exp\theta - \beta\theta^2\exp\theta + \ldots, \quad (36)$$

valid when $\beta \ll 1$, the principal terms in formulas (10), (23), and (34) increase by a factor of $(1 - 6\beta)^{-1}$, while the principal terms in formulas (11), (12), and (24) increase by a factor of $(1 - 6\beta)^{-1/2}$, which for small β is insignificant. With the Shvets method we can find the characteristics of ignition as well for the exact function $\exp(-E/RT)$ in the same manner, for example, as formulas (21) and (22) of [10] were found; however, in this case they are rather cumbersome in form.

In conclusion we note that the formulas which we found for the characteristics of ignition, in view of the assumptions adopted in their derivation, are valid only for moderate values of L, Pr, and Sc and when $L \ll 1$, $\mathrm{Pr} \ll 1$, $\mathrm{Sc} \ll 1$ or $L \gg 1$, $\mathrm{Pr} \gg 1$, $\mathrm{Sc} \gg 1$, they lose significance.

NOTATION

$\theta = (T - T_c)\,E/RT_c^2$ is the dimensionless temperature; E is the activation energy; R is the universal gas constant; T_c is the temperature of the heated catalytic surface; $\theta_0 = (T_c - T_0)\,E/RT_c^2$; T_0 is the initial temperature and the temperature of the reacting liquid outside of the boundary layer; $x = \frac{x_1}{l}$, $y = \frac{\sqrt{\mathrm{Re}}}{l}\int\limits_0^{y_1}\frac{\rho}{\rho_0}\,dy_1$,

$z = \bar{z}\sqrt{\frac{qk_0\rho_0E}{\lambda_0RT_c^2}\exp\left(-\frac{E}{RT_c}\right)}$, $\xi = x_1(g\beta_0RT_c^2/E\nu^2)^{1/3}$,

$\eta = y_1(g\beta_0RT_c^2/E\nu^2)^{1/3}$, are dimensionless coordinates; l is a characteristic dimension; x_1, y_1, $\bar{z}$ are dimensional coordinates; k_0 is a pre-exponent; λ_0, μ_0, and ρ_0 are the coefficients of thermal conductivity, viscosity, and density for $T = T_0$; $\mathrm{Re} = u_\infty l/v$ is the Reynolds number; u_∞ is the unperturbed velocity of the flow; v is the kinematic viscosity; $\tau = qk_0Et/c_pRT_c^2\exp(-E/RT_c)$ is the dimensionless time; t is time; q is the thermal effect of the reaction; τ_* is the heating time; Δ, Δ_1, and Δ_2 are the thicknesses of the hydrodynamic, temperature, and diffusion boundary layers; $\mathrm{Pr} = v/\varkappa$ is the Prandtl number; $\varkappa$ is the coefficient of thermal diffusivity; $L = \varkappa/D$ is the Lewis-Semenov number; D is the coefficient of concentrated diffusion; $\beta = RT_c/E$; $\gamma = c_pRT_c^2/qE$ is a dimensionless parameter; c_p is the heat capacity at constant pressure; $\mathrm{Sc} = v/D$ is the Schmidt number; $a = qEk_0c_0/\lambda_0RT_c^2\,(Ev^2/g\beta_0RT_c^2)^{2/3}\exp(-E/RT_c)$ is a dimensionless parameter; $\alpha = qEk_0c_0lP/RT_c^2u_\infty c_p\exp(-E/RT_c)$ is a dimensionless parameter; $U = v_x(E/g\beta_0vRT_c^2)^{1/3}$; $u = v_x/u_\infty$; v_x is the longitudinal component of flow velocity; β_0 is the coefficient of volumetric expansion; c_0 is the initial concentration;

$$A = -\frac{\theta_0}{\Delta_1}\left(1 + \frac{\Delta_1}{6}\times\frac{d\Delta_1}{d\tau} + \frac{2\Delta_1^3}{\theta_0^3\Delta_2}\right);$$

$$B = \frac{1}{\Delta_2}\left(1 + \frac{L\Delta_2}{6}\,\frac{d\Delta_2}{d\tau} + \frac{2L\gamma\Delta_1^2}{\theta_0^3\Delta_2}\right);$$

$$C = -\frac{\theta_0}{\Delta_1}\left[1 + \frac{\mathrm{Pr}\,\Delta_1^3}{48\sqrt{x}}\times\left(\Delta_1^{-1}\frac{d\Delta_1}{dx} - \frac{1}{4x}\right) - \frac{2\alpha\Delta_1^3}{\theta_0^4\Delta_2}\right];$$

$$F = \frac{1}{\Delta_2}\left[1 + \frac{\mathrm{Sc}\,\Delta_2^3}{48\sqrt{x}}\left(\Delta_2^{-1}\frac{d\Delta_2}{dx} - \frac{1}{4x}\right) + \right.$$

$$\left. + \frac{2a\gamma\,\mathrm{Sc}\,\Delta_1^3}{\mathrm{Pr}\,\theta_0^3\,\Delta_2}\right];$$

$$G = -\frac{\theta_0}{\Delta_1}\left(1 + \frac{2a\,\Delta_1^3}{\Delta_2\theta_0^4} - \frac{\mathrm{Pr}\,\theta_0\Delta_1^3}{48}\,\frac{d\Delta_1}{d\xi}\right);$$

$$H = \frac{1}{\Delta_2}\left\{1 + \frac{\theta_0\,\mathrm{Sc}\,\Delta_2^3}{4}\times\right.$$

$$\times\left[\frac{\Delta_2^2}{45\Delta_1}\left(\frac{1}{\Delta_2}\,\frac{d\Delta_2}{d\xi} - \frac{1}{4\Delta_1}\,\frac{d\Delta_1}{d\xi}\right) - \frac{\Delta_2}{10}\,\frac{d\Delta_2}{d\xi} + \right.$$

$$\left.\left. + \frac{1}{9}\left(\frac{\Delta_1}{\Delta_2}\,\frac{d\Delta_2}{d\xi} + \frac{1}{2}\,\frac{d\Delta_1}{d\xi}\right)\right] + \frac{2a\gamma\,\mathrm{Sc}\Delta_1^3}{\mathrm{Pr}\,\Delta_2\theta_0^3}\right\}.$$

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